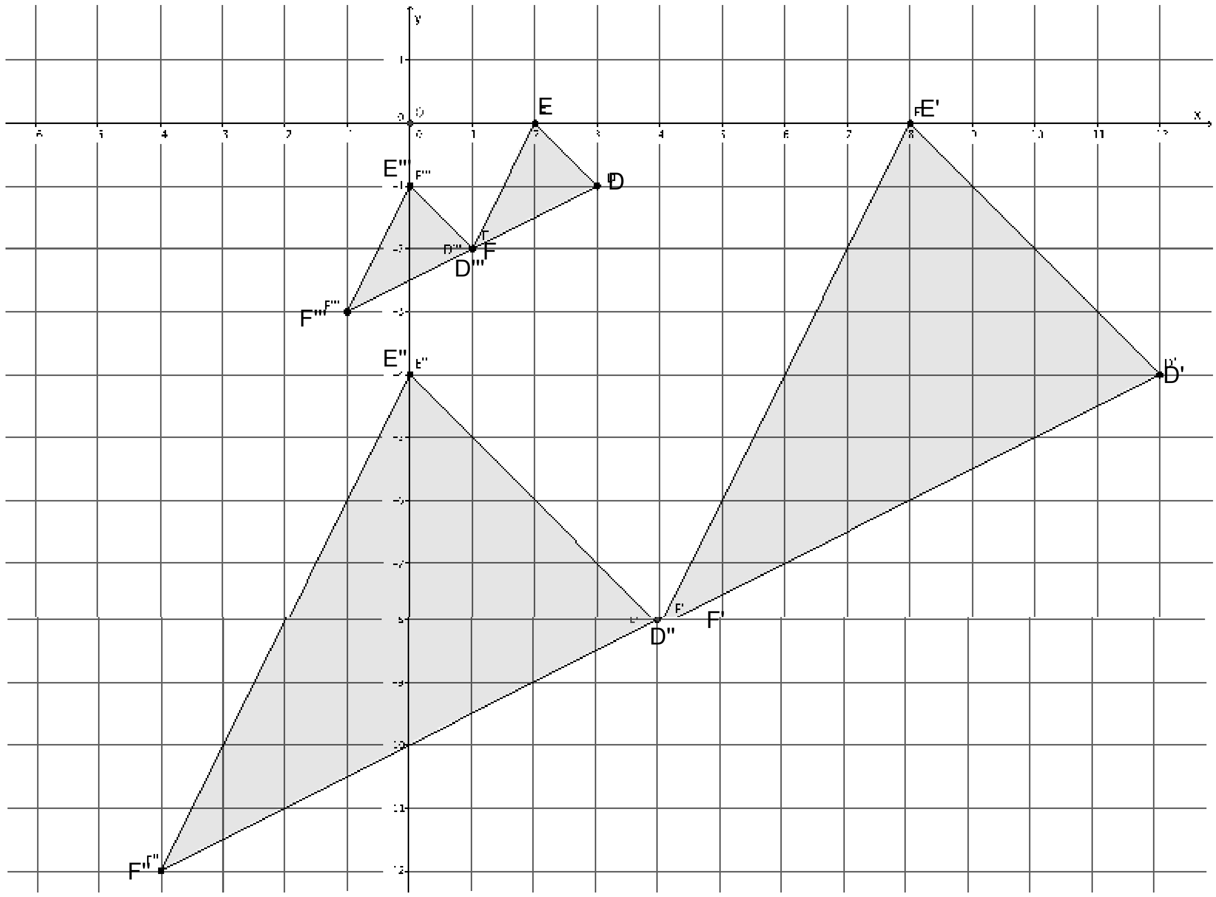
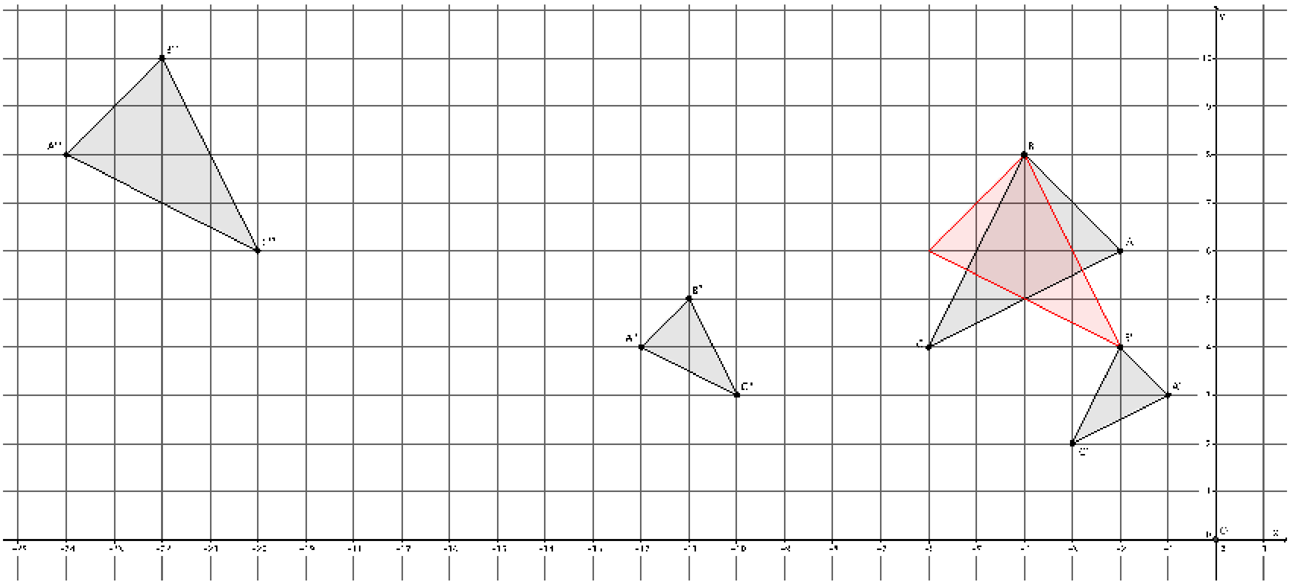
1. In the picture below, we have a triangle $DEF$ that has been dilated from center $O$ by scale factor $r = 4$. It is noted by $D'E'F'$. We also have a triangle $D''E''F''$, which is congruent to triangle $D'E'F'$ (i.e., $\triangle D'E'F' \cong \triangle D''E''F''$). Describe the sequence of a dilation, followed by a congruence (of one or more rigid motions) that would map triangle $D''E''F''$ onto triangle $DEF$.



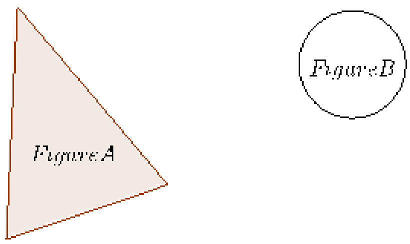

*First, we must dilate triangle $D''E''F''$ by scale factor $r = \frac{1}{4}$ to shrink it to the size of triangle $DEF$. Next, we must translate the dilated triangle, noted by $D'''E'''F'''$, one unit up and two units to the right. This sequence of the dilation followed by the translation would map triangle $D''E''F''$ onto triangle $DEF$.*

2. Triangle $ABC$ was dilated from center $O$ by scale factor $r = \frac{1}{2}$. The dilated triangle is noted by $A'B'C'$. Another triangle $A''B''C''$ is congruent to triangle $A'B'C'$ (i.e., $\triangle A''B''C'' \cong \triangle A'B'C'$). Describe the dilation followed by the basic rigid motions that would map triangle $A''B''C''$ onto triangle $ABC$.



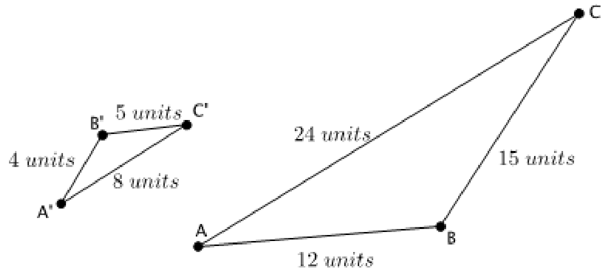
*Triangle $A''B''C''$ will need to be dilated from center $O$ by scale factor $r = 2$ to bring it to the same size as triangle $ABC$. This will produce a triangle noted by $A'''B'''C'''$. Next, triangle $A'''B'''C'''$ will need to be translated 18 units to the right and two units down, producing the triangle shown in red. Next, rotate the red triangle $d$ degrees around point $B$, so that one of the segments of the red triangle coincides completely with segment $BC$. Then, reflect the red triangle across line $BC$. The dilation, followed by the congruence described, will map triangle $A''B''C''$ onto triangle $ABC$.*

3. Are the two figures shown below similar? If so, describe a sequence that would prove the similarity. If not, state how you know they are not similar.



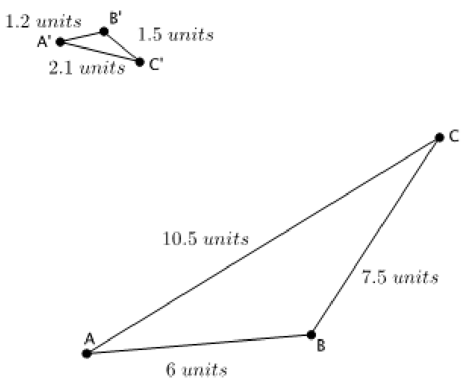

*No, these figures are not similar. There is no single rigid motion, or sequence of rigid motions, that would map Figure $A$ onto Figure $B$.*

4. Triangle $ABC$ is similar to triangle $A'B'C'$ (i.e., $\triangle ABC \sim \triangle A'B'C'$). Prove the similarity by describing a sequence that would map triangle $A'B'C'$ onto triangle $ABC$.



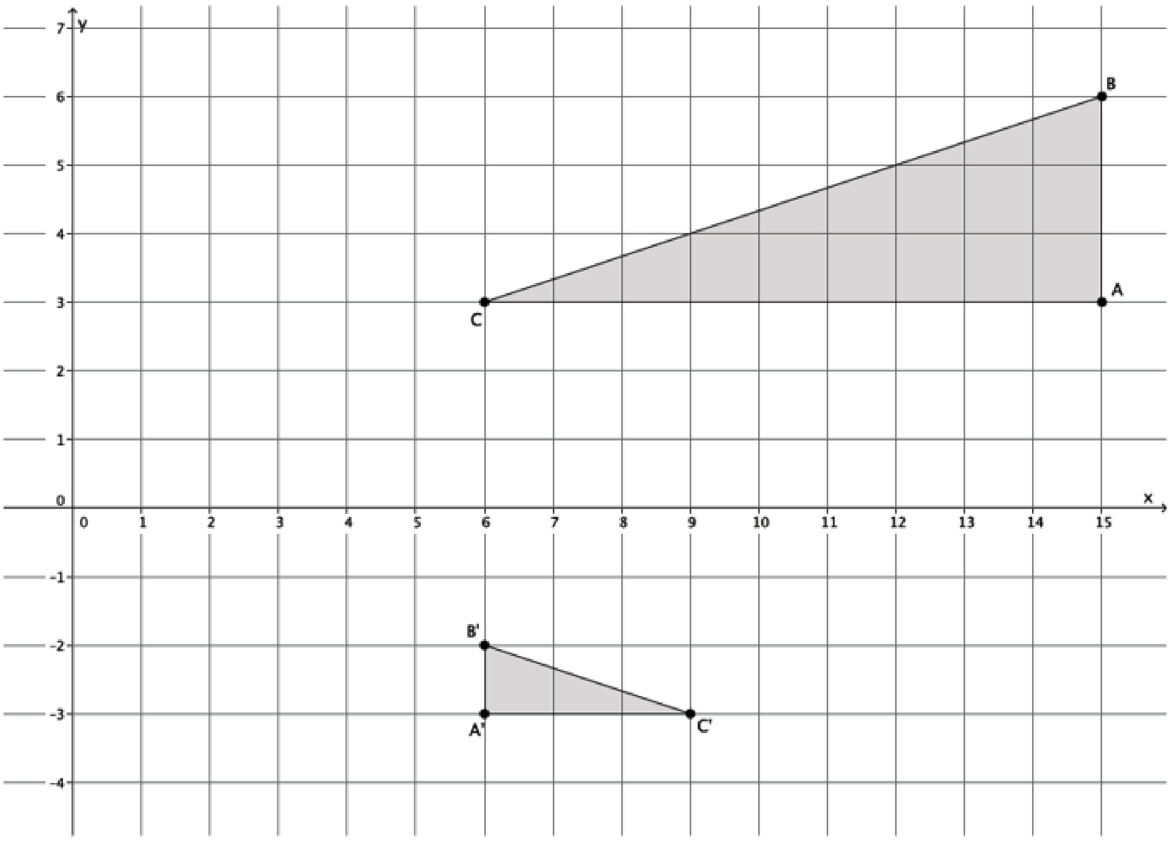

*The scale factor that would magnify triangle $A'B'C'$ to the size of triangle $ABC$ is $r = 3$. The sequence that would prove the similarity of the triangles is a dilation from center $O$ by a scale factor of $r = 3$, followed by a translation along vector $\overrightarrow{A'A}$, and finally, a reflection across line $AC$.*

5. Are the two figures shown below similar? If so, describe a sequence that would prove $\triangle ABC \sim \triangle A'B'C'$. If not, state how you know they are not similar.




*Yes, the triangles are similar. The scale factor that triangle $ABC$ has been dilated is $r = \frac{1}{5}$. The sequence that proves the triangles are similar is as follows: dilate triangle $A'B'C'$ from center $O$ by scale factor $r = 5$, then translate triangle $A'B'C'$ along vector $\overrightarrow{C'C}$; next, rotate triangle $A'B'C'$ $d$ degrees around point $C$; and finally, reflect triangle $A'B'C'$ across line $AC$.*

6. Describe a sequence that would show $\triangle ABC \sim \triangle A'B'C'$.



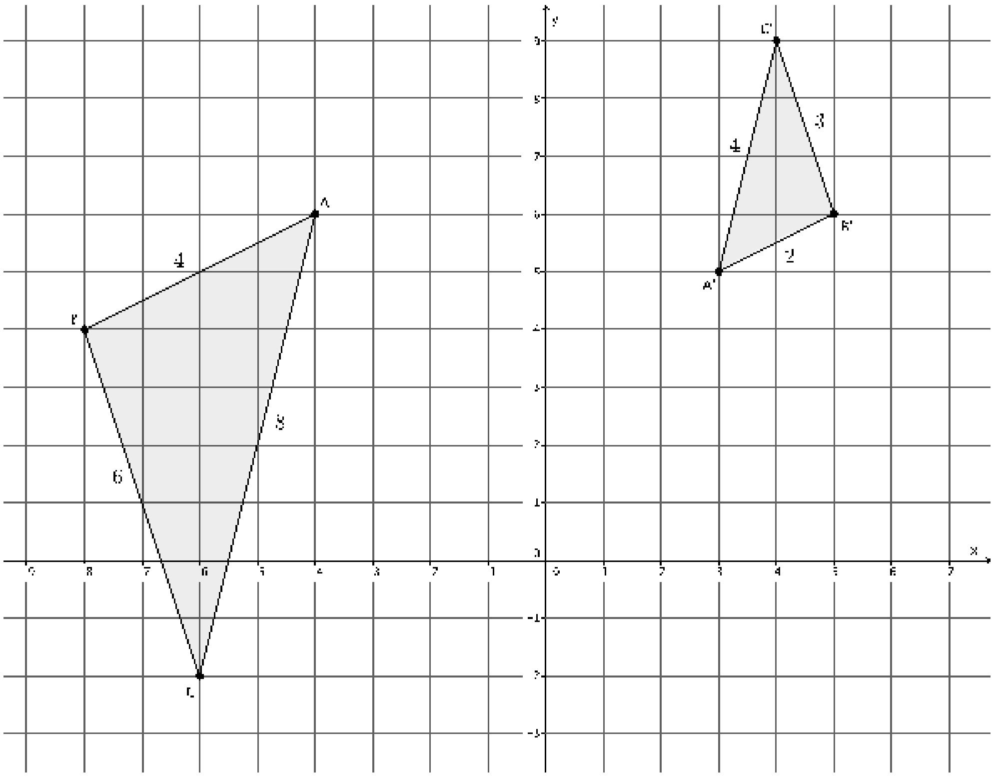

*Since $r|AB| = |A'B'|$, then $r \times 3 = 1$ and $r = \frac{1}{3}$. A dilation from the origin by scale factor $r = \frac{1}{3}$ will make $\triangle ABC$ the same size as $\triangle A'B'C'$. Then, a translation of the dilated image of $\triangle ABC$ four units down and one unit to the right, followed by a reflection across line $A'B'$ will map $\triangle ABC$ onto $\triangle A'B'C'$, proving the triangles to be similar.*

# Lesson 9: Basic Properties of Similarity

## Classwork

### Exploratory Challenge 1

The goal is to show that if $\triangle ABC$ is similar to $\triangle A'B'C'$, then $\triangle A'B'C'$ is similar to $\triangle ABC$. Symbolically, if $\triangle ABC \sim \triangle A'B'C'$, then $\triangle A'B'C' \sim \triangle ABC$.




a. First determine whether or not $\triangle ABC$ is in fact similar to $\triangle A'B'C'$. (If it isn't, then no further work needs to be done.) Use a protractor to verify that the corresponding angles are congruent and that the ratios of the corresponding sides are equal to some scale factor.

Angle A = Angle A'=50 degrees

Angle B = Angle B'=97 degrees

Angle C = Angle C'=33 degrees

$\frac{AB}{A'B'} =$

$\frac{AC}{A'C'} =$

$\frac{BC}{B'C'} =$

b. Describe the sequence of dilation followed by a congruence that proves $\triangle ABC \sim \triangle A'B'C'$.

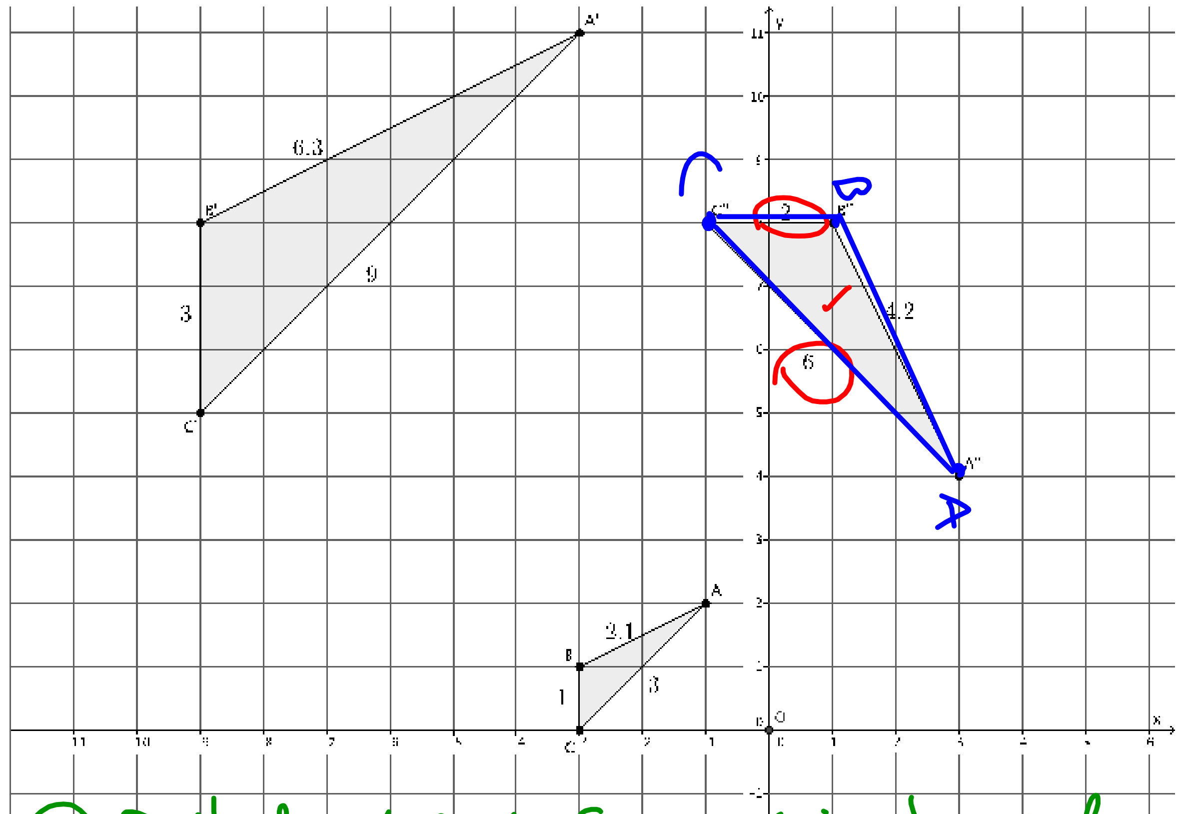
c. Describe the sequence of dilation followed by a congruence that proves $\triangle A'B'C' \sim \triangle ABC$.

d. Is it true that $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle ABC$? Why do you think this is so?

GOAL: $\triangle ABC \sim \triangle A'B'C'$ & $\triangle A'B'C' \sim \triangle A''B''C''$

### Exploratory Challenge 2

The goal is to show that if $\triangle ABC$ is similar to $\triangle A'B'C'$, and $\triangle A'B'C'$ is similar to $\triangle A''B''C''$, then $\triangle ABC$ is similar to $\triangle A''B''C''$. Symbolically, if $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$, then $\triangle ABC \sim \triangle A''B''C''$




① Dilate $\triangle ABC$ from origin by scale factor $r=2$.

② Translate right 5

③ Rotate around pt A until AB lines up with A''B''

a. Describe the similarity that proves $\triangle ABC \sim \triangle A'B'C'$.

b. Describe the similarity that proves $\triangle A'B'C' \sim \triangle A''B''C''$.

c. Verify that, in fact, $\triangle ABC \sim \triangle A''B''C''$ by checking corresponding angles and corresponding side lengths. Then describe the sequence that would prove the similarity $\triangle ABC \sim \triangle A''B''C''$.

Angle A = Angle A'=18 degrees

Angle B = Angle B'=117 degrees

Angle C = Angle C'=45 degrees

$$\frac{AB}{A''B''} = \frac{2.1}{4.2} = 0.5$$

$$\frac{BC}{B''C''} = \frac{1}{2} = 0.5$$

$$\frac{AC}{A''C''} = \frac{3}{6} = 0.5$$

$$\triangle ABC \sim \triangle A''B''C''$$

d. Is it true that if $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$, then $\triangle ABC \sim \triangle A''B''C''$? Why do you think this is so?

Yes, bk of transitive property