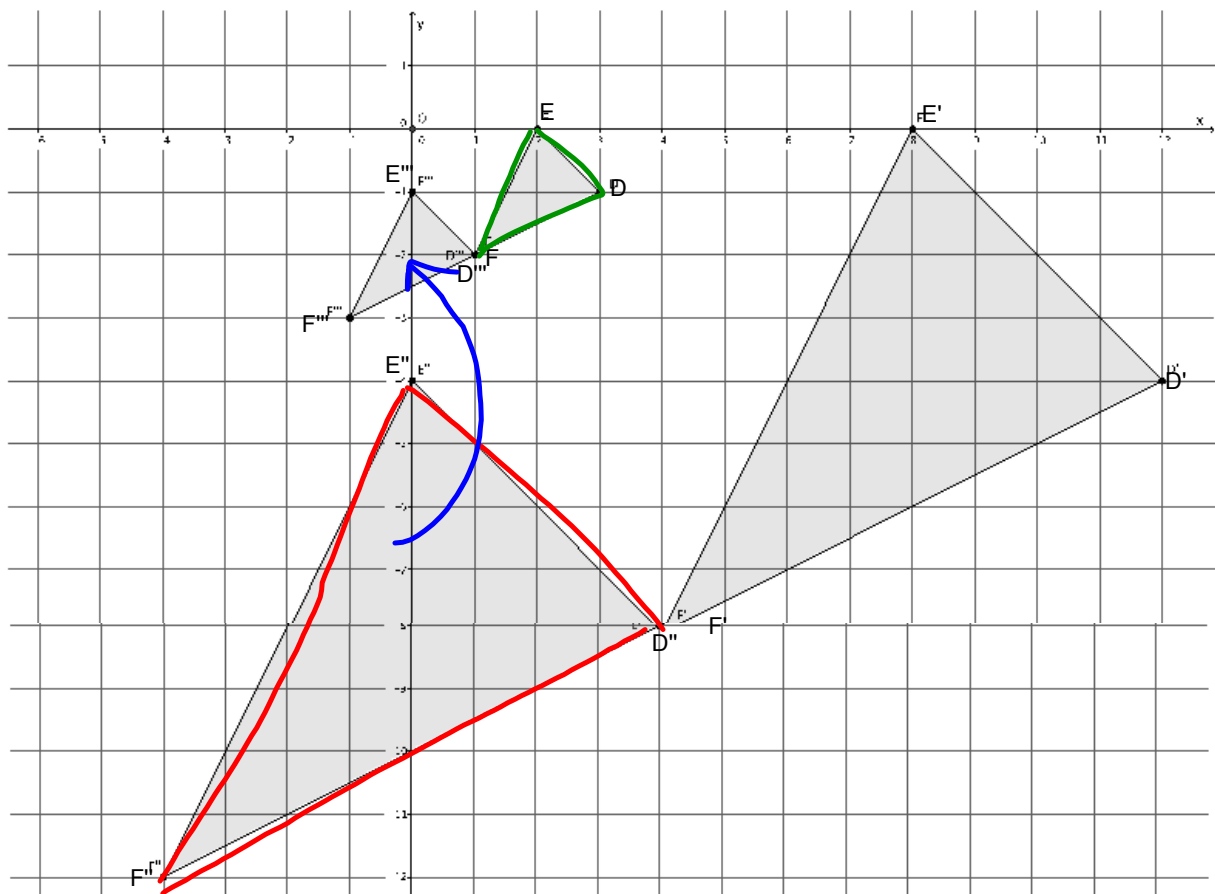
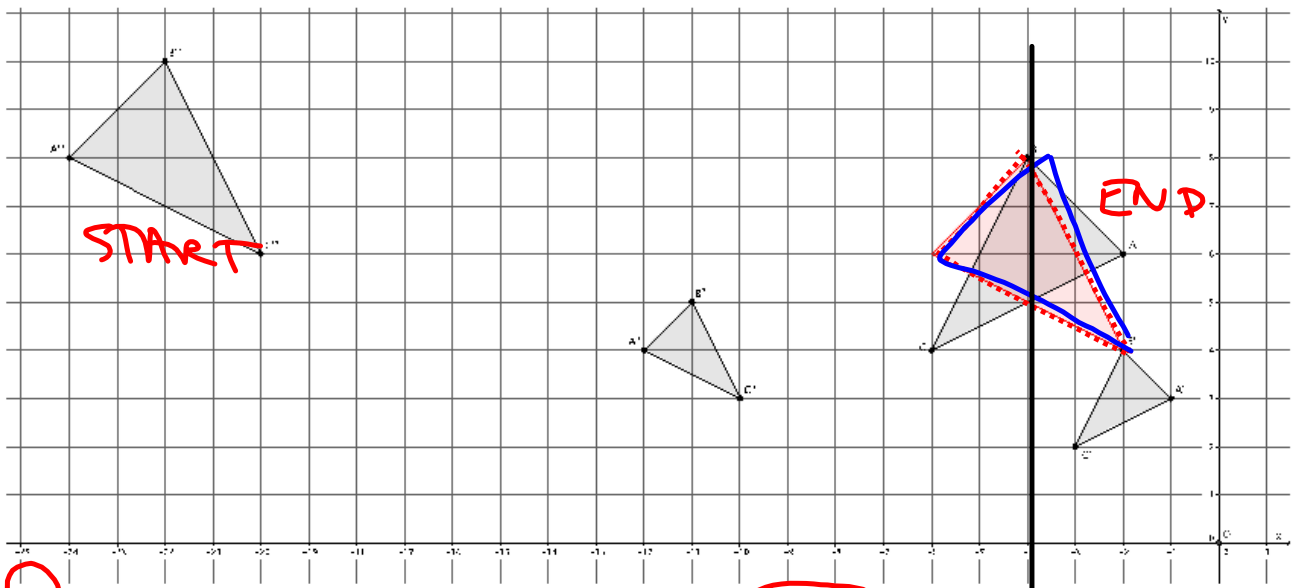


1. In the picture below, we have a triangle DEF that has been dilated from center O by scale factor $r = 4$. It is noted by $D'E'F'$. We also have a triangle $D''E''F''$, which is congruent to triangle $D'E'F'$ (i.e., $\triangle D'E'F' \cong \triangle D''E''F''$). Describe the sequence of a dilation, followed by a congruence (of one or more rigid motions) that would map triangle $D''E''F''$ onto triangle DEF .



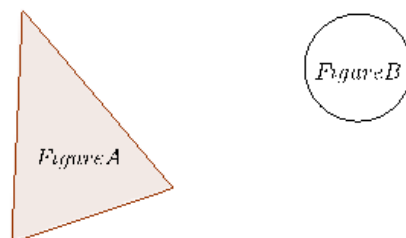
First, we must dilate triangle $D''E''F''$ by scale factor $r = \frac{1}{4}$ to shrink it to the size of triangle DEF . Next, we must translate the dilated triangle, noted by $D'''E'''F'''$, one unit up and two units to the right. This sequence of the dilation followed by the translation would map triangle $D''E''F''$ onto triangle DEF .

2. Triangle ABC was dilated from center O by scale factor $r = \frac{1}{2}$. The dilated triangle is noted by $A'B'C'$. Another triangle $A''B''C''$ is congruent to triangle $A'B'C'$ (i.e., $\triangle A''B''C'' \cong \triangle A'B'C'$). Describe the dilation followed by the basic rigid motions that would map triangle $A''B''C''$ onto triangle ABC .



1 Triangle $A''B''C''$ will need to be dilated from center O by scale factor $r = 2$ to bring it to the same size as triangle ABC . This will produce a triangle noted by $A'''B'''C'''$. Next, triangle $A'''B'''C'''$ will need to be translated 18 units to the right and two units down, producing the triangle shown in red. Next, rotate the red triangle d degrees around point B , so that one of the segments of the red triangle coincides completely with segment BC . Then, reflect the red triangle across line BC . The dilation, followed by the congruence described, will map triangle $A''B''C''$ onto triangle ABC .

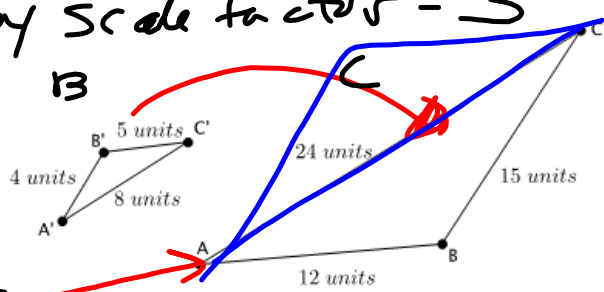
3. Are the two figures shown below similar? If so, describe a sequence that would prove the similarity. If not, state how you know they are not similar.



No, these figures are not similar. There is no single rigid motion, or sequence of rigid motions, that would map Figure A onto Figure B.

4. Triangle ABC is similar to triangle $A'B'C'$ (i.e., $\triangle ABC \sim \triangle A'B'C'$). Prove the similarity by describing a sequence that would map triangle $A'B'C'$ onto triangle ABC .

- ① Dilate by scale factor = 3
- ② Translate along $\vec{AA'}$
- ③ Reflect over AC



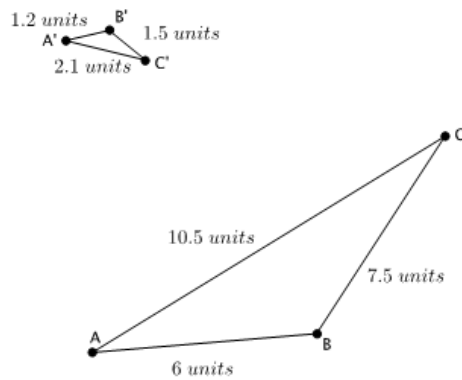
$$\frac{A'C'}{AC} = \frac{8 \div 8}{24 \div 8} = \frac{1}{3}$$

$$\frac{B'C'}{BC} = \frac{5 \div 5}{15 \div 5} = \frac{1}{3}$$

$$\frac{A'B'}{AB} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$$

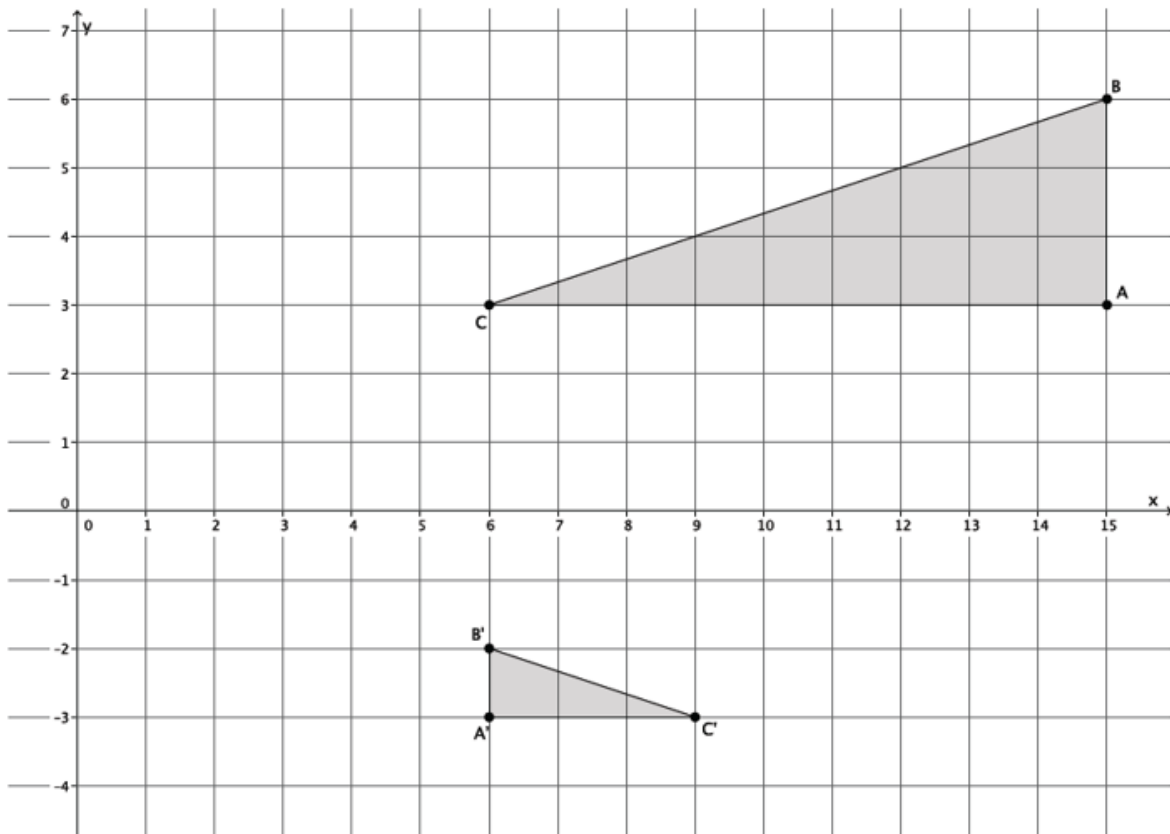
} $r=3$

5. Are the two figures shown below similar? If so, describe a sequence that would prove $\triangle ABC \sim \triangle A'B'C'$. If not, state how you know they are not similar.



Yes, the triangles are similar. The scale factor that triangle ABC has been dilated is $r = \frac{1}{5}$. The sequence that proves the triangles are similar is as follows: dilate triangle $A'B'C'$ from center O by scale factor $r = 5$, then translate triangle $A'B'C'$ along vector $\vec{C'C}$; next, rotate triangle $A'B'C'$ d degrees around point C ; and finally, reflect triangle $A'B'C'$ across line AC .

6. Describe a sequence that would show $\triangle ABC \sim \triangle A'B'C'$.



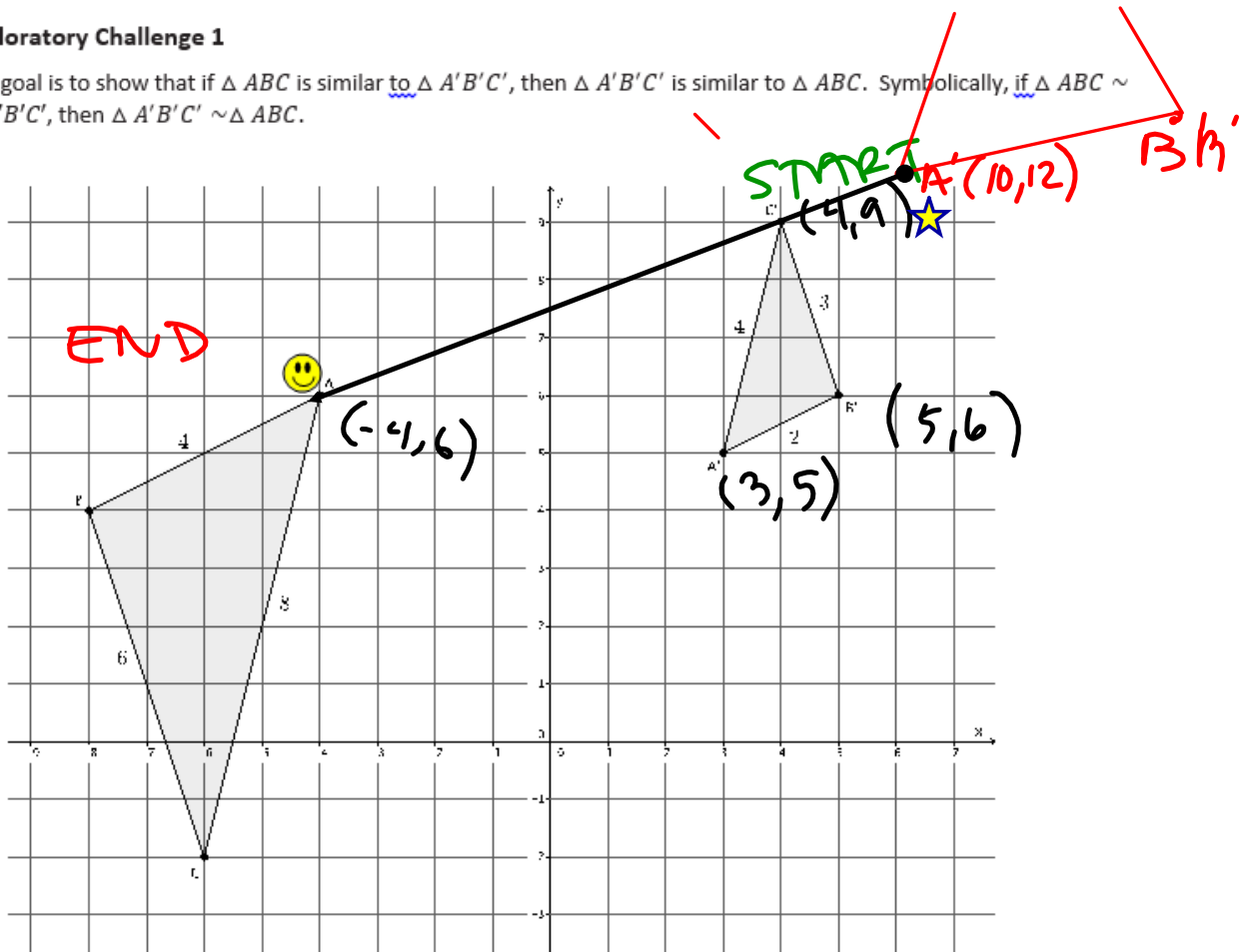
Since $r|AB| = |A'B'|$, then $r \times 3 = 1$ and $r = \frac{1}{3}$. A dilation from the origin by scale factor $r = \frac{1}{3}$ will make $\triangle ABC$ the same size as $\triangle A'B'C'$. Then, a translation of the dilated image of $\triangle ABC$ four units down and one unit to the right, followed by a reflection across line $A'B'$ will map $\triangle ABC$ onto $\triangle A'B'C'$, proving the triangles to be similar.

Lesson 9: Basic Properties of Similarity

Classwork

Exploratory Challenge 1

The goal is to show that if $\triangle ABC$ is similar to $\triangle A'B'C'$, then $\triangle A'B'C'$ is similar to $\triangle ABC$. Symbolically, if $\triangle ABC \sim \triangle A'B'C'$, then $\triangle A'B'C' \sim \triangle ABC$.



- a. First determine whether or not $\triangle ABC$ is in fact similar to $\triangle A'B'C'$. (If it isn't, then no further work needs to be done.) Use a protractor to verify that the corresponding angles are congruent and that the ratios of the corresponding sides are equal to some scale factor.

Angle A = Angle A' = 50 degrees

Angle B = Angle B' = 97 degrees

Angle C = Angle C' = 33 degrees

$$\frac{AB}{A'B'} = \frac{4}{2} = 2$$

$$\frac{AC}{A'C'} = \frac{8}{4} = 2$$

$$\frac{BC}{B'C'} = \frac{6}{3} = 2$$

b. Describe the sequence of dilation followed by a congruence that proves $\triangle ABC \sim \triangle A'B'C'$.

① Dilate from the center
by scale factor $r = \frac{1}{2}$.

② Translate up 2 right 5

③ Rotate 180°

OR
Rotate until AC matches
with A'C'

c. Describe the sequence of dilation followed by a congruence that proves $\triangle A'B'C' \sim \triangle ABC$.

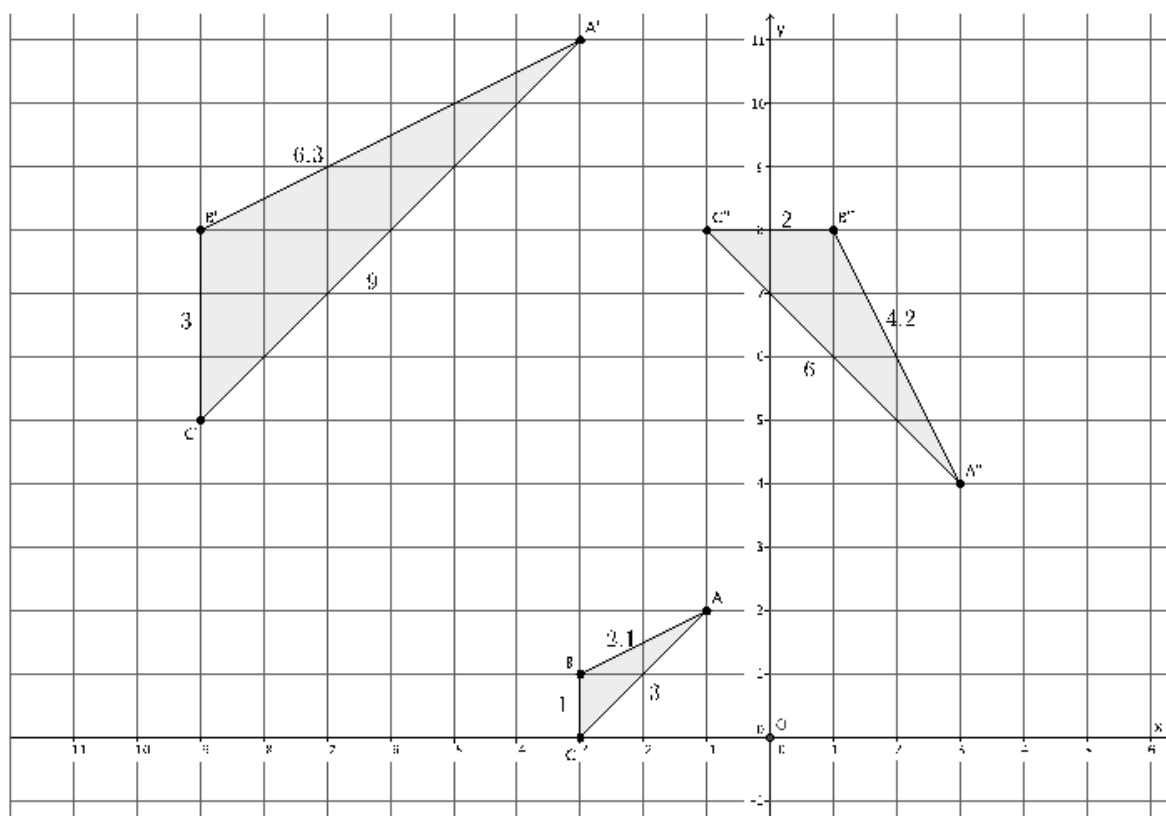
- ① Dilate from origin by a factor of $r=2$.
- ② Translate Down 6, left 14
- ③ Rotate until AC matches up with $A'C'$

d. Is it true that $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle ABC$? Why do you think this is so?

Yes! Because of symmetric property.

Exploratory Challenge 2

The goal is to show that if $\triangle ABC$ is similar to $\triangle A'B'C'$, and $\triangle A'B'C'$ is similar to $\triangle A''B''C''$, then $\triangle ABC$ is similar to $\triangle A''B''C''$. Symbolically, if $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$, then $\triangle ABC \sim \triangle A''B''C''$.



- a. Describe the similarity that proves $\triangle ABC \sim \triangle A'B'C'$.

b. Describe the similarity that proves $\Delta A'B'C' \sim \Delta A''B''C''$.

- c. Verify that, in fact, $\triangle ABC \sim \triangle A''B''C''$ by checking corresponding angles and corresponding side lengths. Then describe the sequence that would prove the similarity $\triangle ABC \sim \triangle A''B''C''$.



- d. Is it true that if $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$, then $\triangle ABC \sim \triangle A''B''C''$? Why do you think this is so?

