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Lesson 4: Fundamental Theorem of Similarity (FTS)

**Fundamental Theorem of Similarity Investigation**

**GOAL:** Students experimentally verify the properties related to the Fundamental Theorem of Similarity (FTS).

Students will need:

* piece of lined paper
* centimeter ruler
* protractor
* calculator

*Background:* The last few days we have focused on dilation. We now want to use what we know about dilation to come to some conclusions about the concept of similarity in general.

A regular piece of notebook paper can be a great tool for discussing similarity. What do you notice about the lines on the notebook paper?

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Keep that information in mind as we proceed through this activity.

1. On the first line of your paper, mark a point $O$. We will use this as our center.
2. Mark the point $P$ a few lines down from the center $O$. From point $O$, draw a ray $\vec{OP}$. Now, choose a $P'$ farther down the ray, also on one of the lines of the notebook paper. For example, you may have placed point $P$ three lines down from the center, and point $P'$ five lines down from the center.
3. Recall, by definition of dilation: $\left|OP'\right|=r\left|OP\right|$*.* Solve this equation for *r*.
4. In my example, the scale factor $r=\frac{5}{3}$ because $OP'$ is $5$ lines from point *O*, and $OP$is $3$ lines down. On the top of your lined paper, write down the scale factor that you have obtained.
5. Now draw another ray, $\vec{OQ}$. Use the same scale factor to mark points $Q$ and $Q'$. In my example, I would place $Q$ three lines down, and $Q'$ five lines down from the center.
6. Now connect point $P$ to point $Q$ and point $P'$ to point $Q'$. What do you notice about lines $PQ$and $P'Q'$?
7. Use your protractor to measure angles $∠OPQ$ and $∠OP'Q'$. What do you notice and why is it so? (Think about your answer to #6, and what kind of angles $∠OPQ$ and $∠OP'Q'$ are.)
8. Now, without using your protractor, what can you say about angles $∠OQP$ and $∠OQ'P'$?
9. Use your centimeter ruler to measure the lengths of segments $OP$and $OP'$*.*

$OP=\\_\\_\\_\\_\\_\\_\\_cm$$OP'$*=\_\_\_\_\_\_cm*

1. By definition of dilation, we expect $\left|OP'\right|=r\left|OP\right|$ (that is, we expect the length of segment $OP'$ to be equal to the scale factor times the length of segment $OP)$. Verify that this is true.
2. Use your centimeter ruler to measure the lengths of segments $OQ$ and $OQ'$.

$OQ=\\_\\_\\_\\_\\_\\_\\_cm$$OQ'$*=\_\_\_\_\_\_cm*

1. We expect $\left|OQ'\right|=r\left|OQ\right|$ (that is, we expect the length of segment $OQ'$ to be equal to the scale factor times the length of segment$OQ)$. Verify that this is true.
2. Bear in mind that we have dilated points $P$and $Q$ from center $O$along their respective rays. Do you expect the segments $PQ$and $P'Q'$ to have the relationship $\left|P'Q'\right|=r\left|PQ\right|$?
3. Measure the segments $PQ$and $P'Q'$ to see if they have the relationship $\left|P'Q'\right|=r\left|PQ\right|$.

$P'Q'=\\_\\_\\_\\_\\_\\_\\_cm$$PQ$*=\_\_\_\_\_\_cm*

1. Does $\left|P'Q'\right|=r\left|PQ\right|?$ Prove it.

It should be somewhat surprising that, in fact, segments $PQ$and $P'Q'$ enjoy the same properties as the segments that we actually dilated.

1. Now mark a point $A$ on line $PQ$ between points $P$and $Q$.
2. Draw a ray from center $O$ through point $A$ and then mark $A'$ on the line $P'Q'$.
3. Do you think $\left|P'A'\right|=r\left|PA\right|$? Measure the segments and use your calculator to check.
4. Now, mark a point $B$ on the line $PQ$ but this time not on the segment $PQ$ (i.e., not between points $P$ and $Q$).
5. Again, draw the ray from center $O$ through point $B$, and mark the point $B'$ on the line $P'Q'$.
6. Select any segment, $AB$,$PB$,$QB$, and verify that it has the same property as the others.

We have just experimentally verified the properties of the Fundamental Theorem of Similarity (FTS) in terms of dilation. Namely, that the parallel line segments connecting dilated points are related by the same scale factor as the segments that are dilated.

**Theorem (Fundamental Theorem of Similarity):** *Given a dilation with center* $O$ *and scale factor* $r$*, then for any two points* $P$ *and* $Q$ *in the plane so that* $O$*,* $P$*, and* $Q$ *are not collinear, the lines* $PQ$ *and* $P'Q'$ *are parallel, where* $P'=Dilation(P)$ *and* $Q'=Dilation(Q)$*, and furthermore,* $\left|P'Q'\right|=r\left|PQ\right|$*.*

In other words, FTS states that given a dilation from center $O$ and points $P$ and $Q$ (points $O$,$ P$, and$ Q$ are not on the same line):

1. the segments formed when you connect $P$ to $Q$ and $P'$ to $Q'$ are parallel.
2. More surprising is the fact that the segment $PQ$, even though it was not dilated as points $P$ and $Q$ were, dilates to segment $P^{'}Q^{'}$, and the length of segment $P'Q'$ is the length of segment $PQ$ multiplied by the scale factor.

Classwork

Exercise

In the diagram below, points $R$ and $S$ have been dilated from center $O$ by a scale factor of $r=3.$



* 1. If the length of $\left|OR\right|=2.3$ cm, what is the length of $\left|OR^{'}\right|$?
	2. If the length of $\left|OS\right|=3.5$ cm, what is the length of $\left|OS^{'}\right|$?
	3. Connect the point $R$ to the point $S$ and the point $R'$ to the point $S'$. What do you know about lines $RS$ and $R^{'}S^{'}$?
	4. What is the relationship between the length of segment $RS$ and the length of segment $R^{'}S^{'}$?
	5. Identify pairs of angles that are equal in measure. How do you know they are equal?
1. Caleb sketched the following diagram on graph paper. He dilated points $B$ and $C$ from center $O.$



* 1. What is the scale factor $r$? Show your work.
	2. Verify the scale factor with a different set of segments.
	3. Which segments are parallel? How do you know?
	4. Which angles are equal in measure? How do you know?
1. Points $B$ and $C$ were dilated from center $O$.



* 1. What is the scale factor $r$? Show your work.
	2. If the length of $\left|OB\right|=5$, what is the length of $\left|OB^{'}\right|$?
	3. How does the perimeter of triangle $OBC$ compare to the perimeter of triangle $OB'C'$?
	4. Did the perimeter of triangle $OB'C'=r×($perimeter of triangle $OBC)$? Explain.