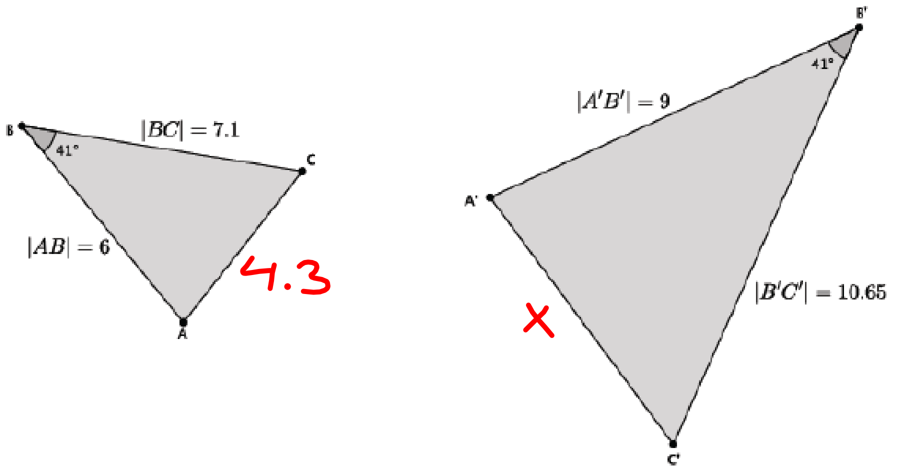
1. In the diagram below, you have $\triangle ABC$ and $\triangle A'B'C'$. Use this information to answer parts (a)–(b).



a. Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.

Yes, $\triangle ABC \sim \triangle A'B'C'$. Since there is only information about one pair of corresponding angles being equal, then the corresponding sides must be checked to see if their ratios are equal.

$$\frac{B'C'}{BC} = \frac{A'B'}{AB} \qquad \frac{10.65}{7.1} = \frac{9}{6}$$

$$63.9 = 63.9$$

Since the cross products are equal, the triangles are similar.

b. Assume the length of side AC is 4.3. What is the length of side $A'C'$?

Let x represent the length of $A'C'$.

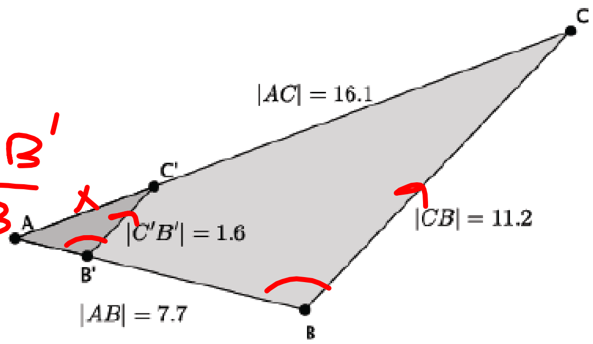
$$\frac{A'C'}{AC} = \frac{A'B'}{AB} \qquad \frac{x}{4.3} = \frac{9}{6} \qquad \frac{6x}{6} = \frac{38.7}{6}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $6x = 38.7$, and $x = 6.45$. The length of side $A'C'$ is 6.45.

$$\boxed{x = 6.45}$$

2. In the diagram below, you have $\triangle ABC$ and $\triangle AB'C'$. Use this information to answer parts (a)–(d).

$$\frac{AC'}{AC} = \frac{C'B'}{CB}$$



a. Based on the information given, is $\triangle ABC \sim \triangle AB'C'$? Explain.

Only $\angle A \cong \angle A$

There is not enough information provided to determine if the triangles are similar. We would need information about a pair of corresponding angles or more information about the side lengths of each of the triangles.

b. Assume line BC is parallel to line $B'C'$. With this information, can you say that $\triangle ABC \sim \triangle AB'C'$? Explain.

If line BC is parallel to line $B'C'$, then $\triangle ABC \sim \triangle AB'C'$. Both triangles share $\angle A$. Another pair of equal angles is $\angle AB'C'$ and $\angle ABC$. They are equal because they are corresponding angles of parallel lines. By the AA criterion, $\triangle ABC \sim \triangle AB'C'$.

$\angle A \cong \angle A$
$\angle AB'C' \cong \angle ABC$, so the triangles are similar

c. Given that $\triangle ABC \sim \triangle AB'C'$, determine the length of side AC'.

Let x represent the length of AC'.

$$\frac{11.2x}{11.2} = \frac{25.76}{11.2}$$

$$\frac{x}{16.1} = \frac{1.6}{11.2}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $11.2x = 25.76$, and $x = 2.3$. The length of AC' is 2.3. $x = 2.3$

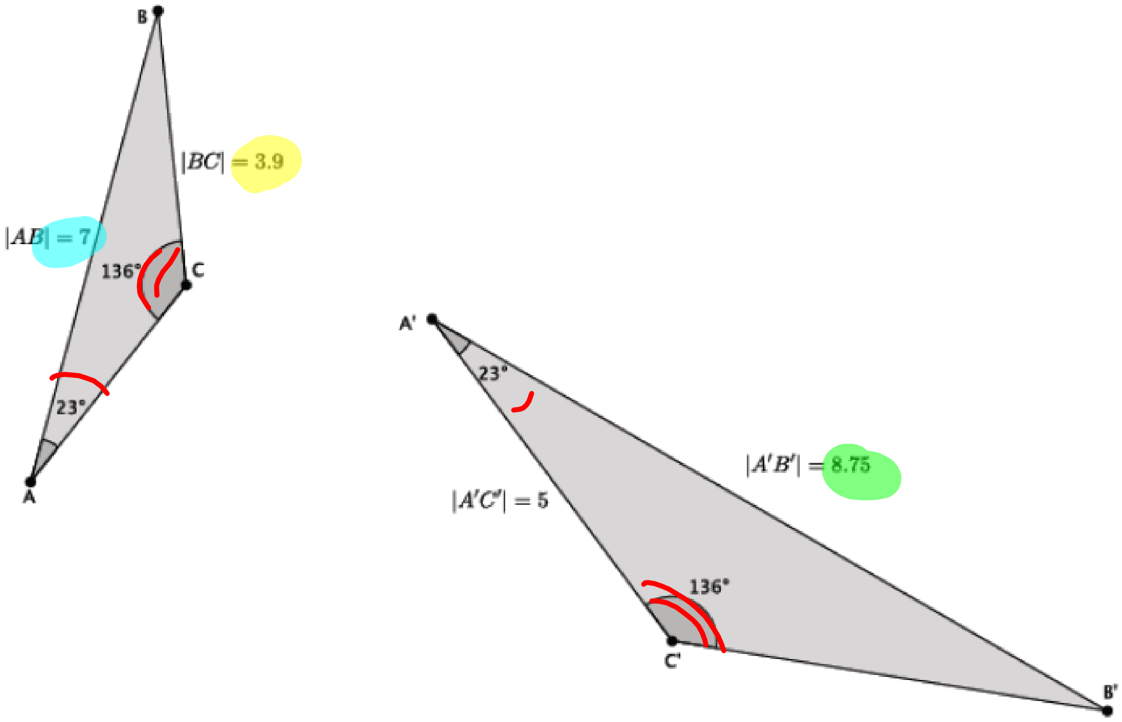
d. Given that $\triangle ABC \sim \triangle AB'C'$, determine the length of side AB'.

Let y represent the length of AB'.

$$\frac{y}{7.7} = \frac{1.6}{11.2}$$

We are looking for the value of y that makes the fractions equivalent. Therefore, $11.2y = 12.32$, and $y = 1.1$. The length of side AB' is 1.1.

3. In the diagram below, you have $\triangle ABC$ and $\triangle A'B'C'$. Use this information to answer parts (a)–(c).



a. Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.

Yes, $\triangle ABC \sim \triangle A'B'C'$. There are two pairs of corresponding angles that are equal in measure. Namely, $\angle A = \angle A' = 23°$, and $\angle C = \angle C' = 136°$. By the AA criterion, these triangles are similar.

b. Given that $\triangle ABC \sim \triangle A'B'C'$, determine the length of side $B'C'$.

Let x represent the length of $B'C'$.

$$\frac{B'C'}{BC} = \frac{A'B'}{AB}$$

$$\frac{x}{3.9} = \frac{8.75}{7}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $7x = 34.125$, and $x = 4.875$. The length of side $B'C'$ is 4.875.

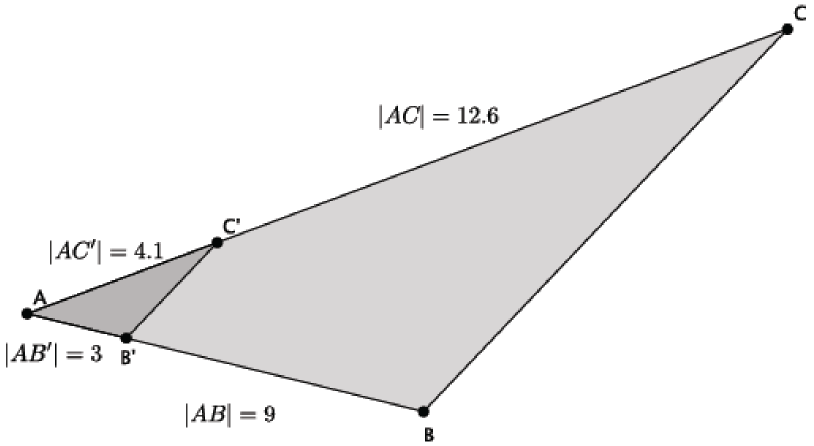
c. Given that $\triangle ABC \sim \triangle A'B'C'$, determine the length of side AC.

Let y represent the length of side AC.

$$\frac{5}{y} = \frac{8.75}{7}$$

We are looking for the value of y that makes the fractions equivalent. Therefore, $8.75y = 35$, and $y =$ The length of side AC is 4.

4. In the diagram below, you have $\triangle ABC$ and $\triangle AB'C'$. Use this information to answer the question below.



Based on the information given, is $\triangle ABC \sim \triangle AB'C'$? Explain.

No, $\triangle ABC$ is not similar to $\triangle AB'C'$. Since there is only information about one pair of corresponding angles, then we must check to see that the corresponding sides have equal ratios. That is, the following must be true:

$$\frac{AB}{AB'} = \frac{AC}{AC'}$$

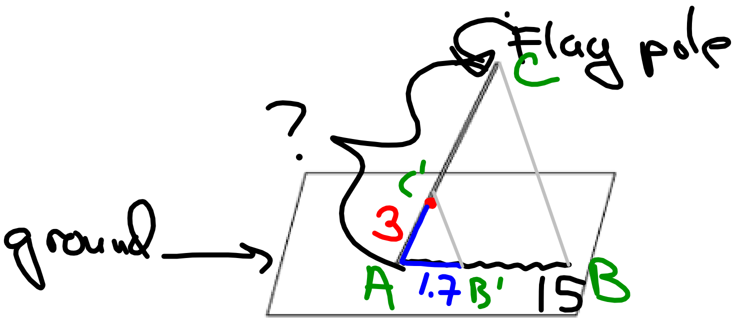
$$\frac{9}{3} \times \frac{12.6}{4.1}. \quad 36.9 \neq 37.8$$

When we compare products of each numerator with the denominator of the other fraction, we see that $36.9 \neq 37.8$. Since the corresponding sides do not have equal ratios, the fractions are not equivalent, and the triangles are not similar.

Classwork

Example 1

Not all flagpoles are perfectly *upright* (i.e., perpendicular to the ground). Some are oblique (i.e., neither parallel nor at a right angle, slanted). Imagine an oblique flagpole in front of an abandoned building. The question is, can we use sunlight and shadows to determine the length of the flagpole?



Assume that we know the following information. The length of the shadow of the flagpole is 15 feet. There is a mark on the flagpole 3 feet from its base. The shadow of this three feet portion of the flagpole is 1.7 feet.

let $x = AC$ = ht of flagpole

$$\frac{AC'}{AC} = \frac{AB'}{AB} \Rightarrow \frac{3}{x} = \frac{1.7}{15}$$

$$3 \cdot 15 = 1.7x$$

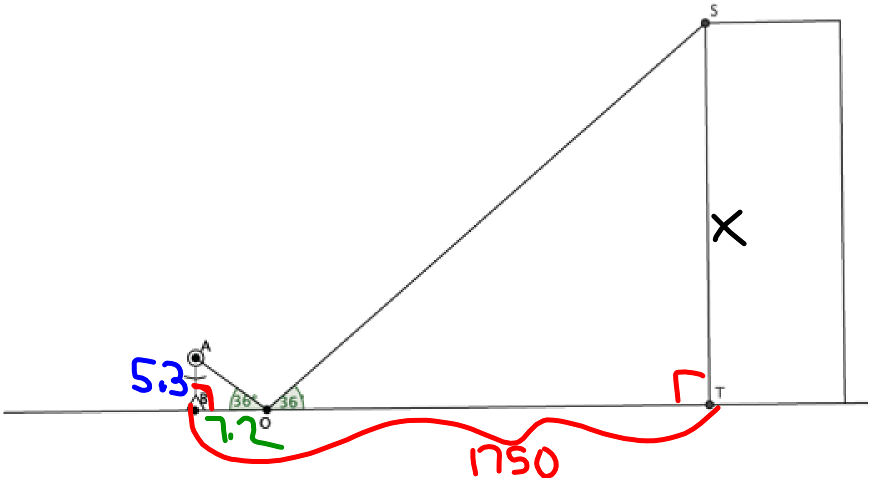
$$\frac{45}{1.7} = \frac{1.7x}{1.7}$$

$$\boxed{26.5 = x}$$

The flagpole is 26.5 ft.

Mathematical Modeling Exercises

1. You want to determine the approximate height of one of the tallest buildings in the city. You are told that if you place a mirror some distance from yourself so that you can see the top of the building in the mirror, then you can indirectly measure the height using similar triangles. Let O be the location of the mirror so that the person shown can see the top of the building.



a. Explain why $\triangle ABO \sim \triangle STO$.

$\angle AOB \cong \angle SOT$ and $\angle ABO \cong \angle STO$

(AKA, the AA criterion)

b. Label the diagram with the following information: The distance from eye-level straight down to the ground is 5.3 feet. The distance from the person to the mirror is 7.2 feet. The distance from the person to the base of the building is 1,750 feet. The height of the building will be represented by x.

17

c. What is the distance from the mirror to the building?

$1750 - 7.2 = 1742.8\text{ft}$

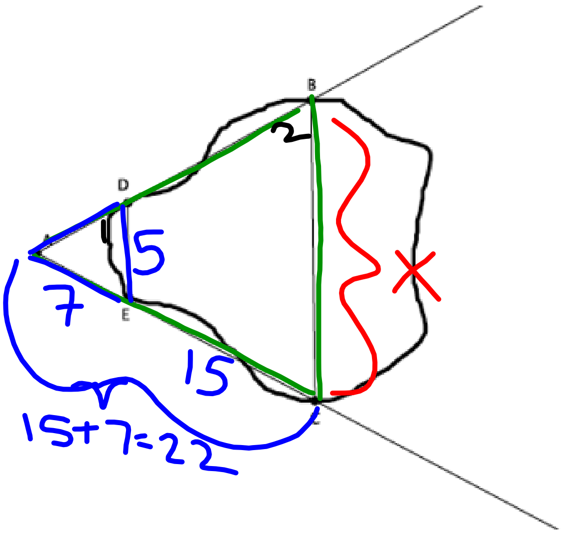
d. Do you have enough information to determine the approximate height of the building? If yes, determine the approximate height of the building. If not, what additional information is needed?

$$\frac{AB}{ST} = \frac{BO}{TO} \Rightarrow \frac{5.3}{x} = \frac{7.2}{1742.8}$$

$$\frac{7.2x}{7.2} = \frac{9236.84}{7.2}$$

$$\boxed{x = 1289.9\text{ft}}$$

2. A geologist wants to determine the distance across the widest part of a nearby lake. The geologist marked off specific points around the lake so that line DE would be parallel to line BC. The segment BC is selected specifically because it is the widest part of the lake. The segment DE is selected specifically because it was a short enough distance to easily measure. The geologist sketched the situation as shown below.



a. Has the geologist done enough work so far to use similar triangles to help measure the widest part of the lake? Explain.

∠A≅∠A and ∠1≅∠2, so yes, by AA criterion.

b. The geologist has made the following measurements: $|DE| = 5$ feet, $|AE| = 7$ feet, and $|EC| = 15$ feet. Does she have enough information to complete the task? If so, determine the length across the widest part of the lake. If not, state what additional information is needed.

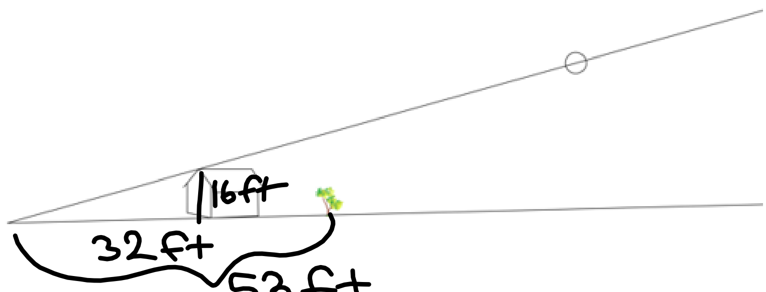
$$\frac{22}{7} = \frac{x}{5}$$

$$\frac{110}{7} = \frac{7x}{7} \qquad x = \frac{110}{7} \approx 15.7\text{ ft}$$

c. Assume the geologist could only measure a maximum distance of 12 feet. Could she still find the distance across the widest part of the lake? What would need to be done differently?

Yes, if she measured AD and AB

3. A tree is planted in the backyard of a house with the hope that one day it will be tall enough to provide shade to cool the house. A sketch of the house, tree, and sun is shown below.



a. What information is needed to determine how tall the tree must be to provide the desired shade?

ht of house
length of shadows

b. Assume that the sun casts a shadow 32 feet long from a point on top of the house to a point in front of the house. The distance from the end of the house's shadow to the base of the tree is 53 feet. If the house is 16 feet tall, how tall must the tree get to provide shade for the house?

$$\frac{32}{53} = \frac{16}{x} \qquad \frac{32x}{32} = \frac{848}{32}$$

$$\boxed{x = 26.5 \text{ ft}}$$

c. Assume that the tree grows at a rate of 2.5 feet per year. If the tree is now 7 feet tall, about how many years will it take for the tree to reach the desired height?

$7 + 2.5 = 9.5$ 1

+2.5	2
12.0	
+2.5	3
14.5	
+2.5	4
17.0	
+2.5	5
19.5	
2.5	6
22.0	
2.5	7
24.5	
+2.5	8
27.0	

$$7 + 2.5t = 26.5$$
$$-7 \qquad -7$$
$$\frac{2.5t}{2.5} = \frac{19.5}{2.5}$$
$$t = 7.8 \text{ or about 8 yrs}$$